Wrocław University of Science and Technology

Soil Mechanics-Lecture IV: Shear strength. Lateral stresses.

HR EXCELLENCE IN RESEARCH

European Funds
Knowledge Education Development

Wrocław University of Science and Technology

European Union
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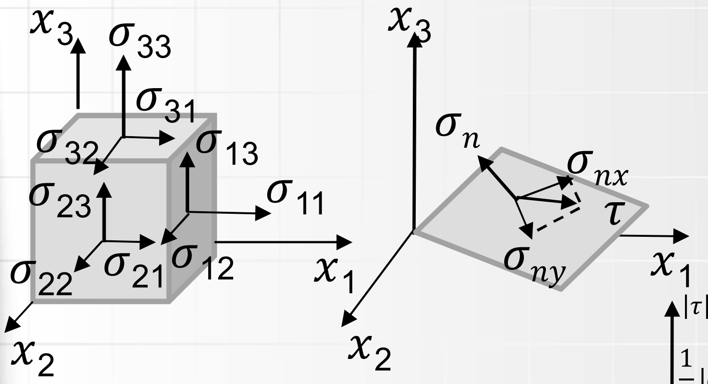
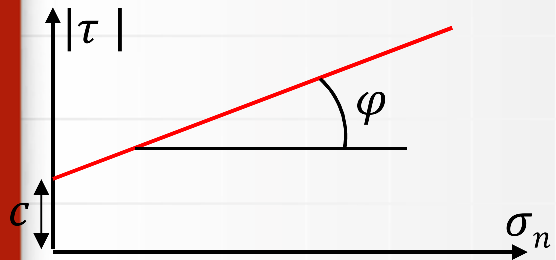
Mohr-Coulomb criterion

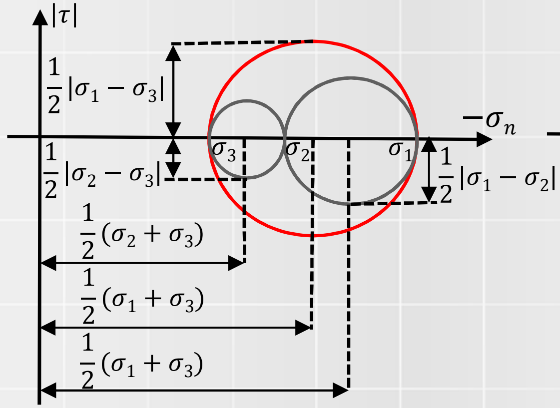
$$\sigma_{11} = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos(2\theta)$$

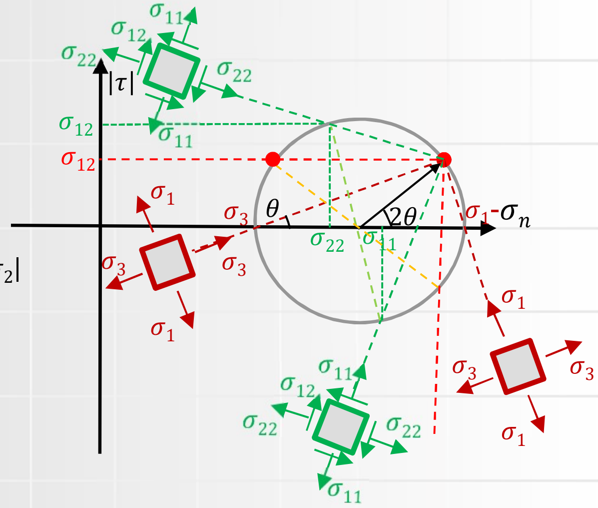
$$\sigma_{22} = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3)\cos(2\theta)$$

$$\sigma_{12} = \sigma_{21} = \frac{1}{2}(\sigma_1 - \sigma_3)\,|\sin(2\theta)|$$



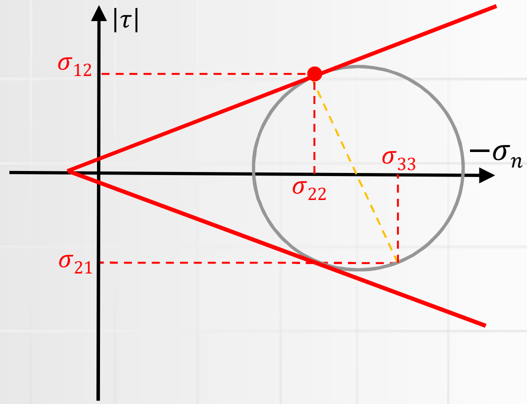




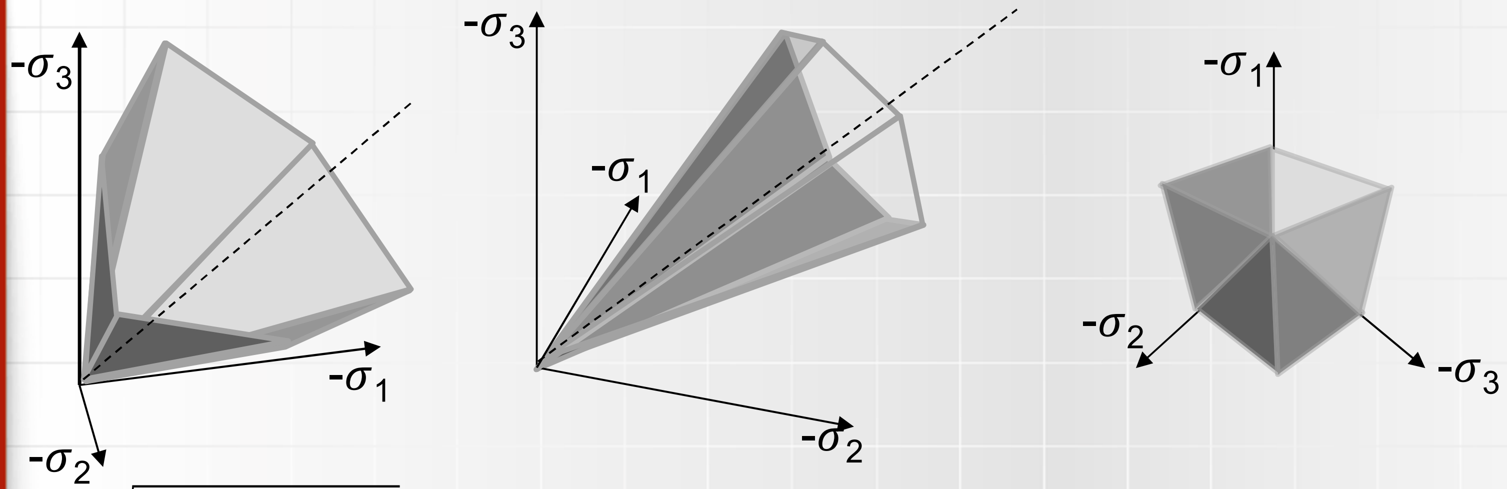


$$\sqrt{\left(\frac{\sigma_{11} - \sigma_{22}}{2}\right)^2 + \sigma_{12}{}^2} + \frac{1}{2}(\sigma_{11} + \sigma_{22})\sin\varphi - c\cos\varphi \geq 0$$

$$\frac{1}{2}|\sigma_1 - \sigma_3| + \frac{1}{2}(\sigma_{11} + \sigma_{22})\sin\varphi - c\cos\varphi \geq 0$$



Mohr- Coulomb in three dimentions



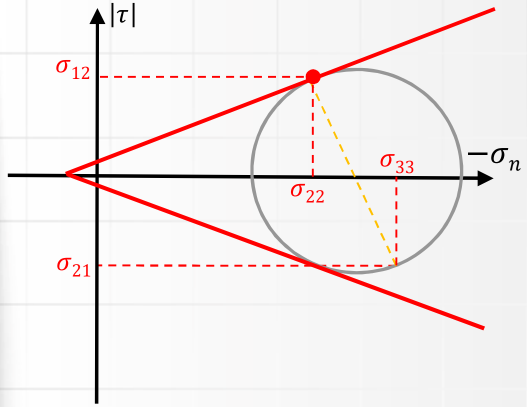
$$\sqrt{\left(\frac{\sigma_{11}-\sigma_{22}}{2}\right)^2+\sigma_{12}{}^2}+\frac{1}{2}(\sigma_{11}+\sigma_{22})\sin\varphi-c\cos\varphi\geq 0$$

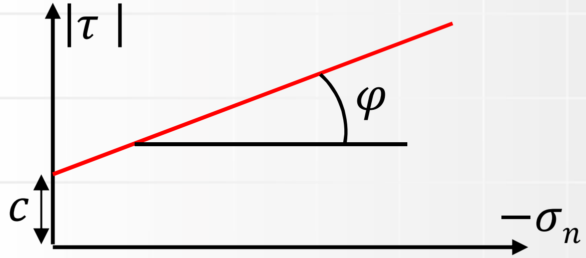
$$\sqrt{\left(\frac{\sigma_{11}-\sigma_{33}}{2}\right)^2+\sigma_{13}{}^2}+\frac{1}{2}(\sigma_{11}+\sigma_{33})\sin\varphi-c\cos\varphi\geq 0$$

$$\sqrt{\left(\frac{\sigma_{22}-\sigma_{33}}{2}\right)^2+\sigma_{23}{}^2}+\frac{1}{2}(\sigma_{22}+\sigma_{33})\sin\varphi-c\cos\varphi\geq 0$$

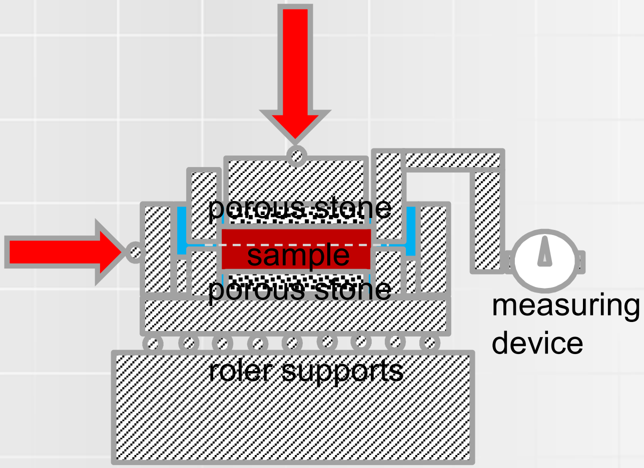
$$f_m(\sigma_{ij})=I_1\sin\phi_m+\frac{1}{2}\left[\ 3(1-\sin\phi_m)\sin\theta+\sqrt{3}(3+\sin\phi_m)\ \cos\theta\right]\sqrt{J_2}-3c\ \cos\phi_m\leq 0 \qquad \theta=\frac{1}{3}\cos^{-1}\left(\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right)$$

Direct shear test





$$|\tau| = \sigma_n \tan\varphi + c$$



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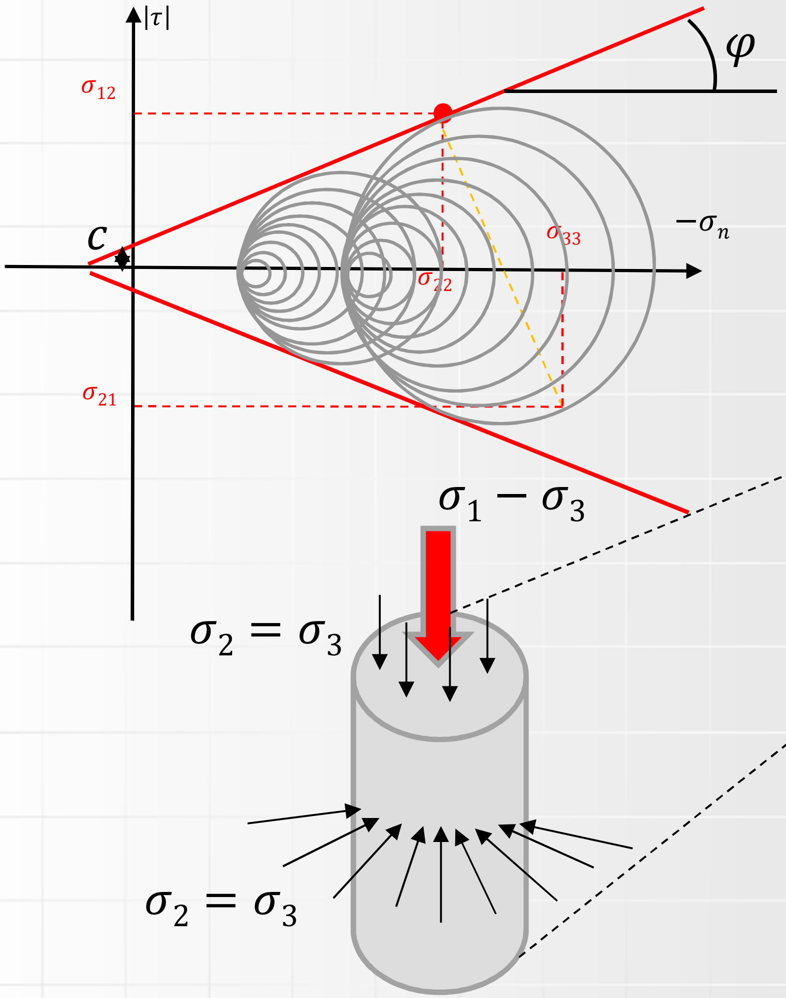
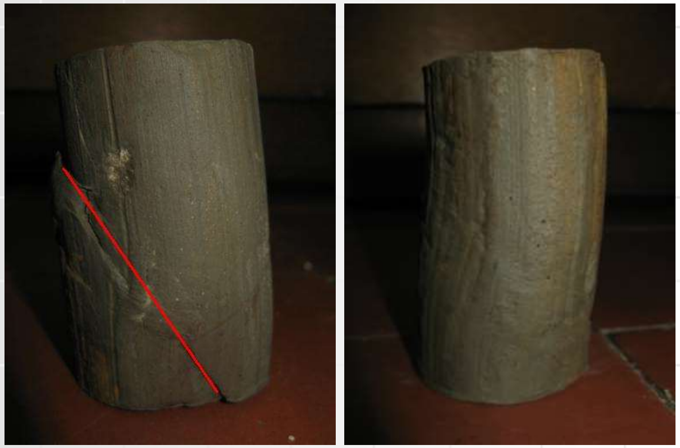
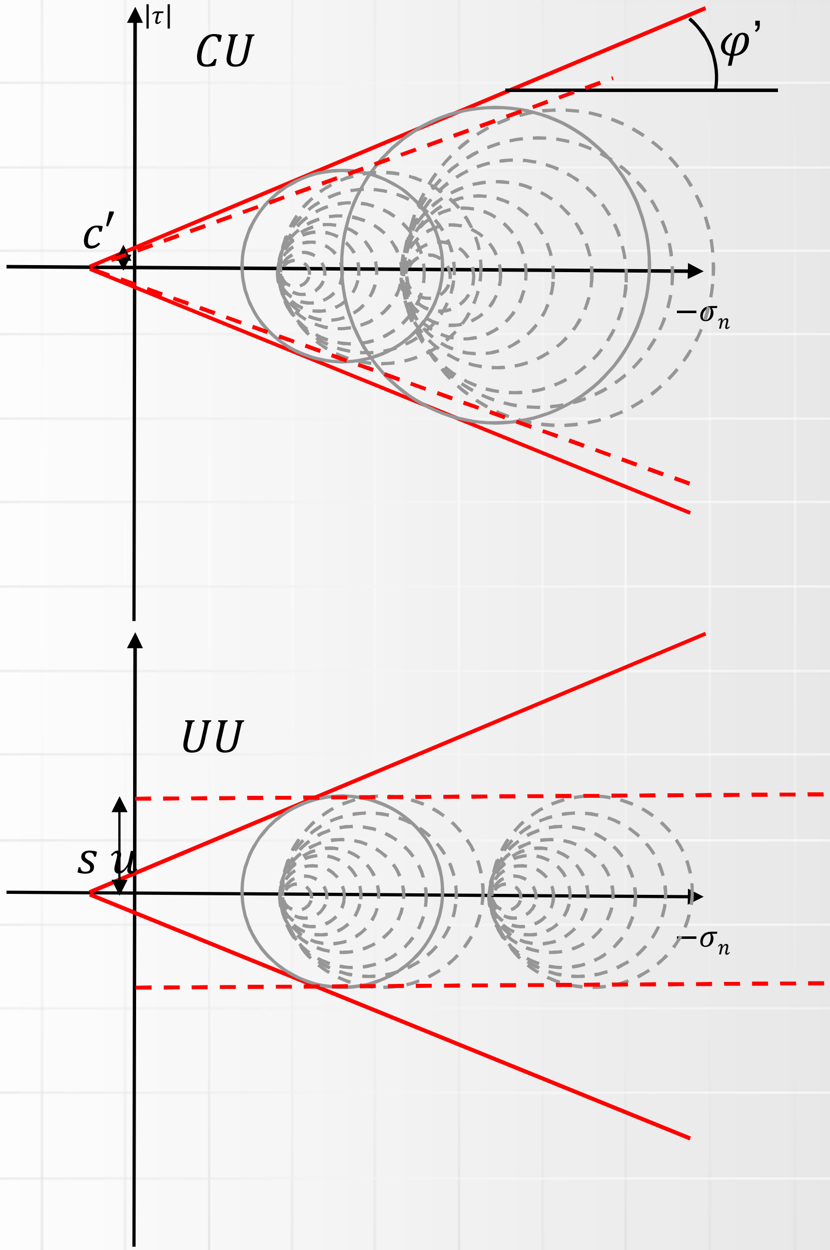
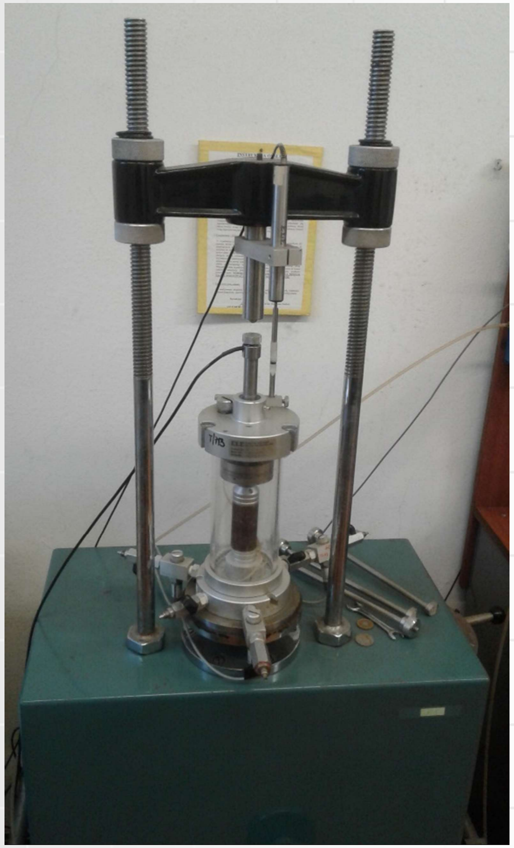
Three axial test

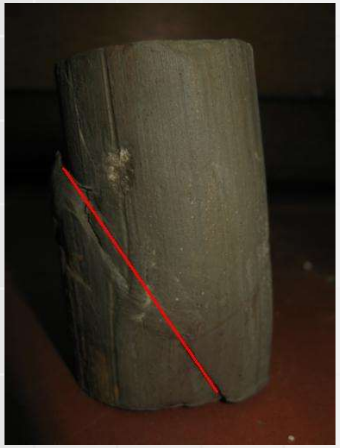




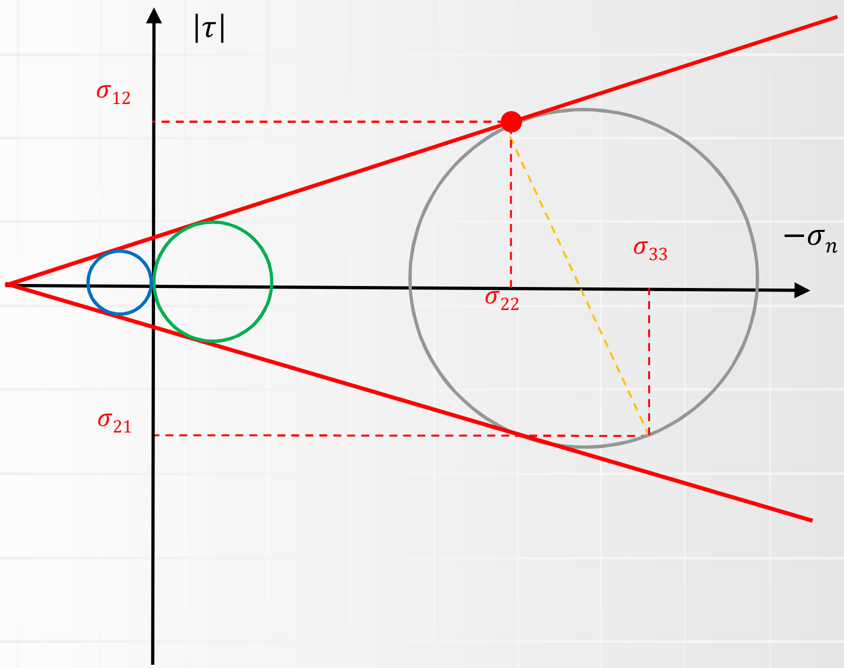
Pore pressure. Undrained behaviour.







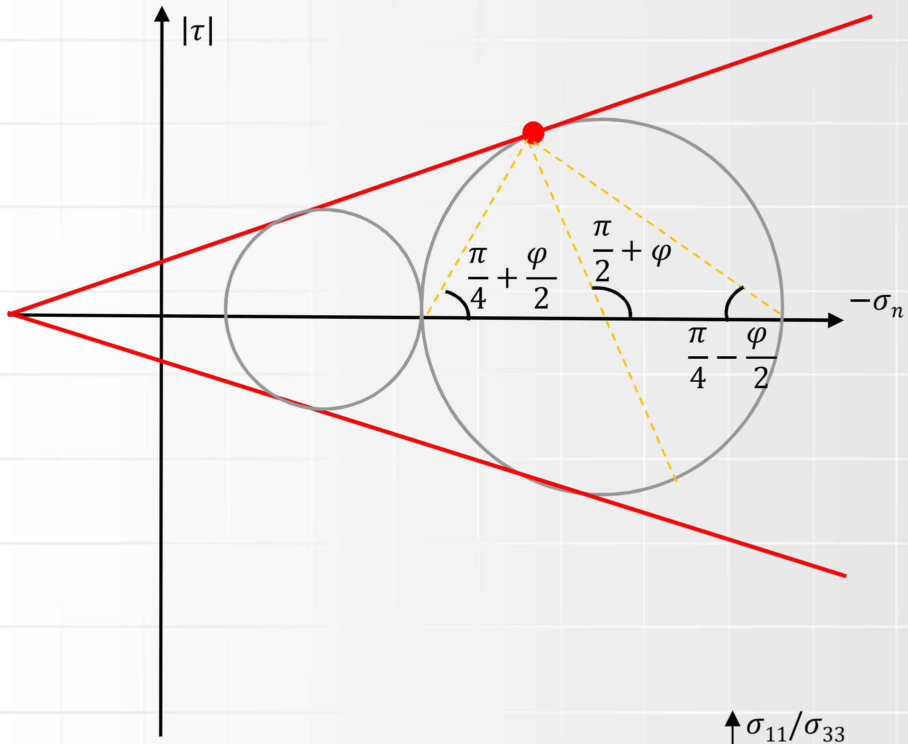
Questions



- Uniaxial compression (and tension) stregth
- Obtain ϕ and c form three axial tests resulst
- Obtain ϕ' and c' from three axial test results
- Obtain ϕ and c by direct shearing test

Coefficents of earth pressure

$\sigma_{22} = \sigma_{11} = \frac{\upsilon}{1-\upsilon}\sigma_{33}$ Earth pressure „at rest” or neutral earth pressure



$$\sigma_3 = \frac{1-\sin\varphi}{1+\sin\varphi}\sigma_1 - 2c\frac{\cos\varphi}{1+\sin\varphi}$$

$$\frac{\cos\varphi}{1+\sin\varphi} = \frac{\sqrt{1-\sin^2\varphi}}{1+\sin\varphi} = \frac{\sqrt{1-\sin\varphi}}{\sqrt{1+\sin\varphi}}$$

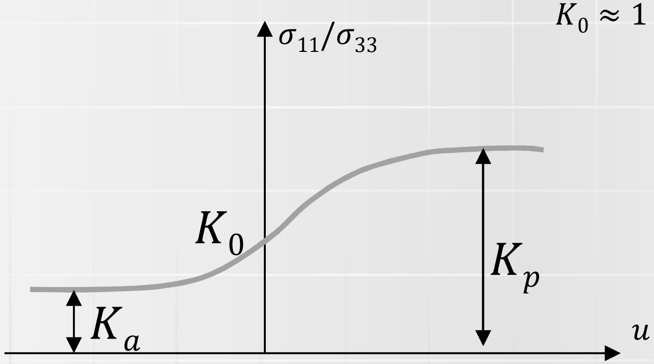
$$K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \tan^2\left(\frac{\pi}{4}-\frac{\varphi}{2}\right)$$ active

$$K_p = \frac{1+\sin\varphi}{1-\sin\varphi} = \tan^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)$$ passive

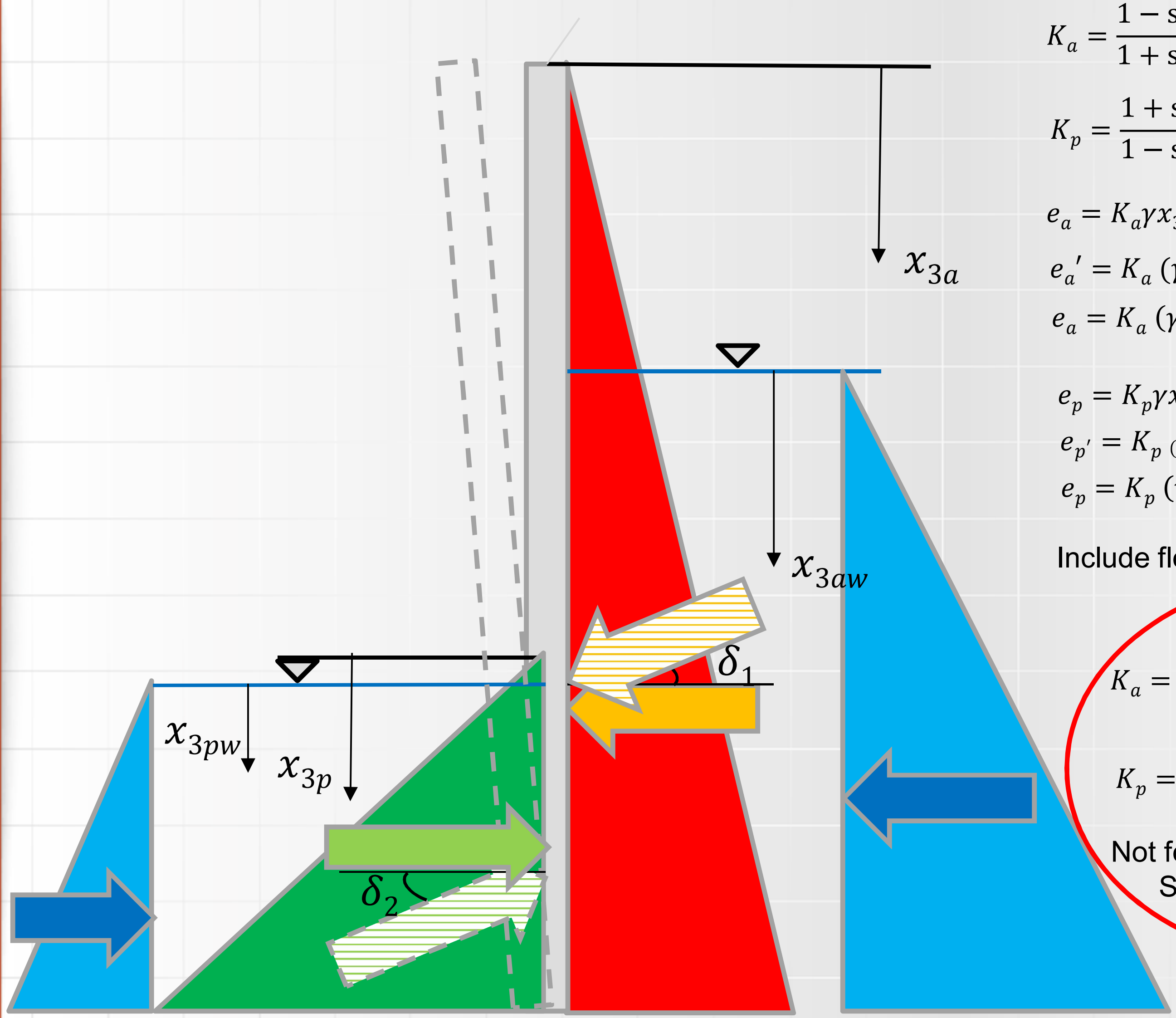
$$\sigma_3 = K_a\sigma_1 - 2c\sqrt{K_a}$$

$$\sigma_1 = K_p\sigma_3 + 2c\sqrt{K_p}$$

$$K_0 \approx 1-\sin\varphi$$



Retaining structures



$$K_a = \frac{1-\sin\varphi}{1+\sin\varphi} = \tan^2\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)$$

$$K_p = \frac{1+\sin\varphi}{1-\sin\varphi} = \tan^2\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$$

$$e_a = K_a \gamma x_3 - 2c\sqrt{K_a}$$

$$e_a{}' = K_a\,(\gamma_{sat} - \gamma_w) x_{3a} - 2c\sqrt{K_a}$$

$$e_a = K_a\,(\gamma_{sat} - \gamma_w) x_{3a} - 2c\sqrt{K_a} + \gamma_w x_{3aw}$$

$$e_p = K_p \gamma x_3 - 2c\sqrt{K_p}$$

$$e_{p'} = K_p\,(\gamma_{sat} - \gamma_w) x_{3p} + 2c\sqrt{K_p}$$

$$e_p = K_p\,(\gamma_{sat} - \gamma_w) x_{3p} + 2c\sqrt{K_p} + \gamma_w x_{3pw}$$

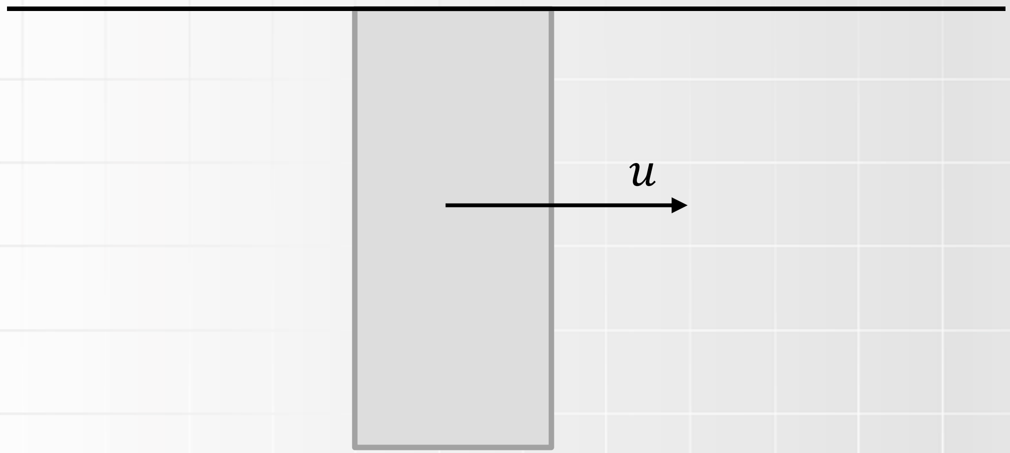
Include flow?

$$K_a = \frac{\cos^2\varphi\,\sec\delta}{\left(1 + \sqrt{\sec\delta\,\sin\varphi\,\sin(\varphi+\delta)}\right)^2}$$

$$K_p = \frac{\cos^2\varphi\,\sec\delta}{\left(1 - \sqrt{\sec\delta\,\sin\varphi\,\sin(\varphi-\delta)}\right)^2}$$

Not for sheet pile wall. Dangerous!
Some experience required!

Questions



•Draw the distribution of earth pressure

•What value of coefficient of lateral earth pressure is assumed in elastic problems

•In which situations lateral active and passive pressures should be used

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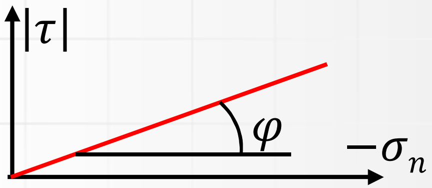
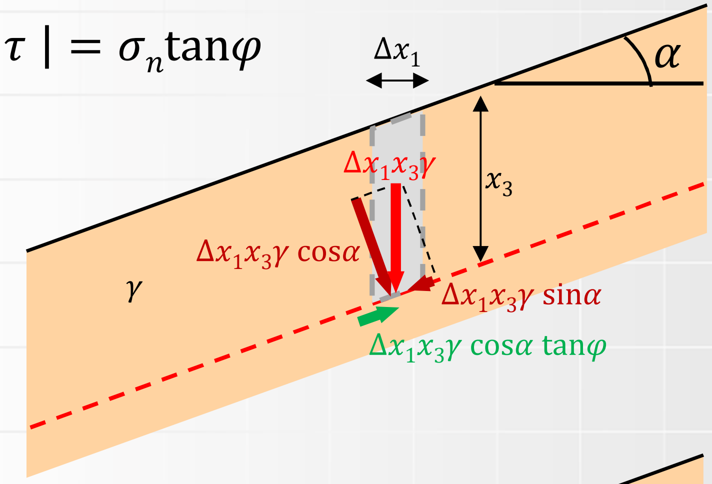
Limit equlibrium methods. Inifinite slope

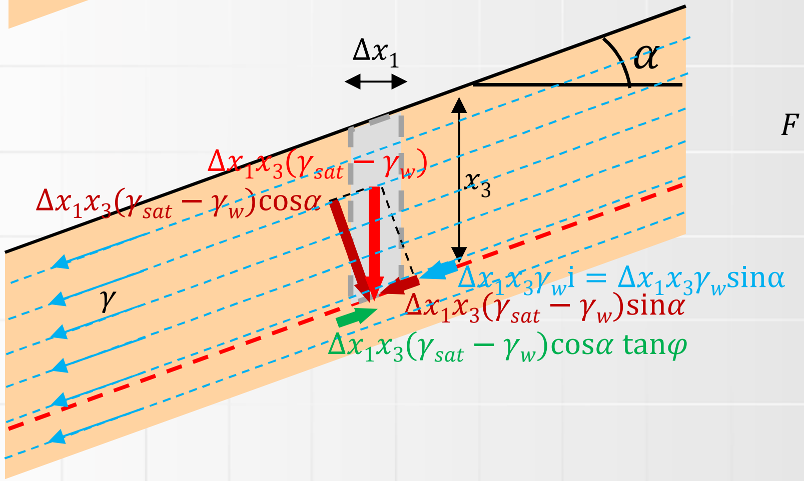


$$|\tau| = \sigma_n \tan\varphi$$

$$F = \frac{stabilizing\ forces}{destabilizing\ forces}$$

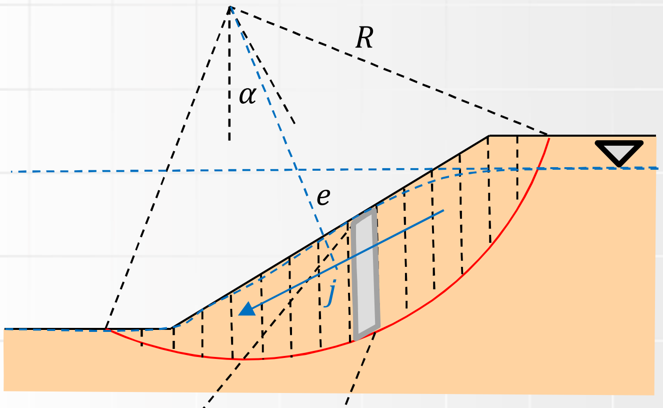


$$F = \frac{\Delta x_1 x_3 \gamma \cos\alpha \tan\varphi}{\Delta x_1 x_3 \gamma \sin\alpha} = \frac{\tan\varphi}{\tan\alpha}$$



$$F = \frac{\Delta x_1 x_3 (\gamma_{sat} - \gamma_w) \cos\alpha \tan\varphi}{\Delta x_1 x_3 (\gamma_{sat} - \gamma_w + \gamma_w) \sin\alpha} = \frac{(\gamma_{sat} - \gamma_w)}{\gamma_{sat}} \frac{\tan\varphi}{\tan\alpha}$$

LEM. Slope stability. Fellenius. Bishop.



Fellenius

for identical b

$$\sum \gamma b R h \sin\alpha = \sum \tau b R \frac{1}{\cos\alpha}$$

$$\tau = \frac{1}{F}(c + \sigma_n{}' \tan\varphi)$$

$$F = \frac{\sum[(c + \sigma_n{}'\tan\varphi)/\cos\alpha]}{\sum \gamma h \sin\alpha}$$

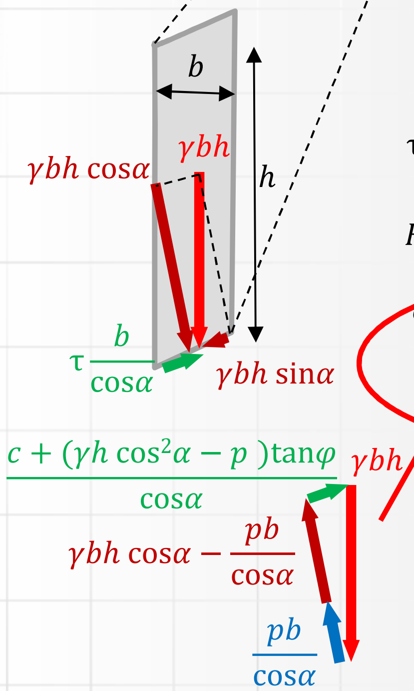
$$\sigma_n{}' = \gamma h \cos^2\alpha - p$$

$$F = \frac{\sum \frac{c + (\gamma h \cos^2\alpha - p)\tan\varphi}{\cos\alpha}}{\sum \gamma h \sin\alpha}$$

equilibrium of a strip not satisfied if there is no horizontal forces!

$$F = \frac{\sum \frac{c + (\gamma_{sat} - \gamma_w) h \cos^2\alpha \tan\varphi}{\cos\alpha}}{\sum(\gamma_{sat} - \gamma_w) h \sin\alpha + j\frac{e}{R}}$$

seem to be reasonable but the equilibrium is still not satisfied!



$$\gamma h = \sigma_n{}' + p + \tau\frac{\sin\alpha}{\cos\alpha}$$

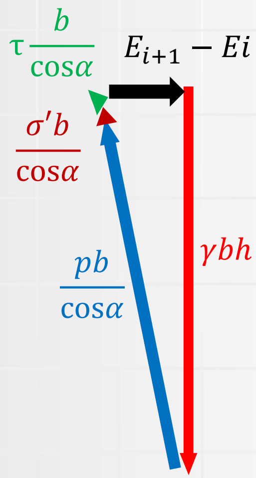
$$\tau = \frac{1}{F}(c + \sigma_n{}'\tan\varphi)$$

$$\sigma_n{}'\left(1 + \frac{\tan\alpha \tan\varphi}{F}\right) = \gamma h - p - \frac{c}{F}\tan\alpha$$

$$F = \frac{\sum[(c + \sigma_n{}'\tan\varphi)/\cos\alpha]}{\sum \gamma h \sin\alpha}$$

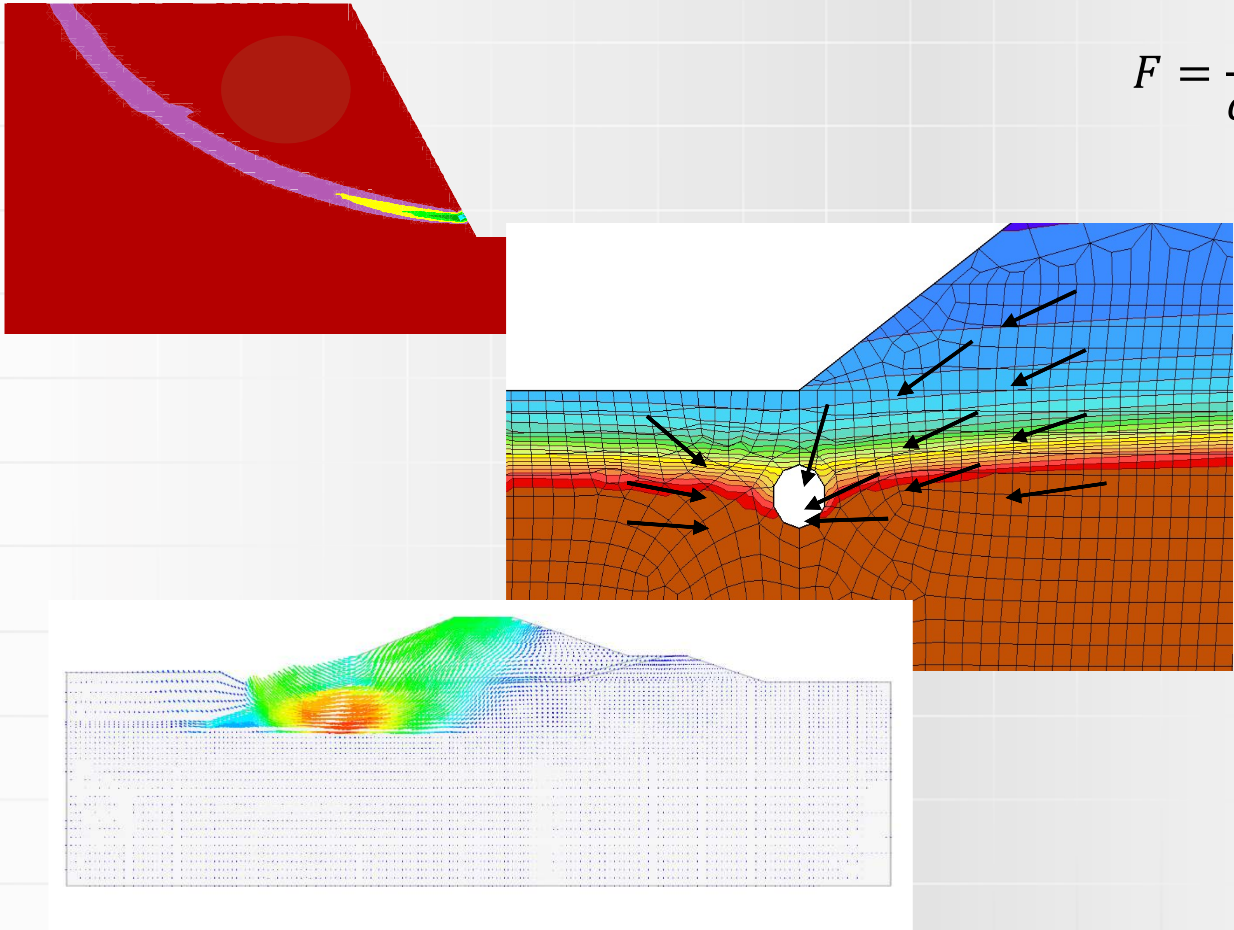
$$F = \frac{\sum \frac{c + (\gamma h - p)\tan\varphi}{\cos\alpha\,(1 + \tan\alpha \tan\varphi / F)}}{\sum \gamma h \sin\alpha}$$

iterative method



Numerical (FEM, FDM) slope stability. Shear Strength Reduction method.

$$F = \frac{c}{c_{crit}} = \frac{\varphi}{\varphi_{crit}}$$



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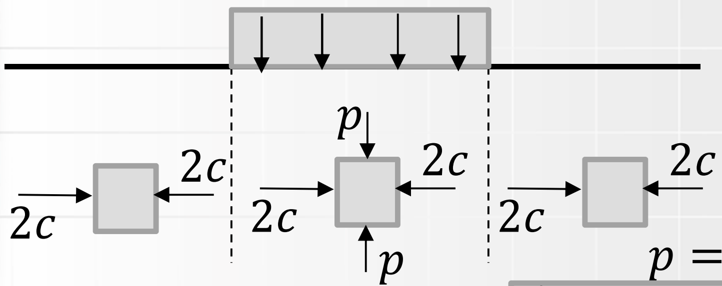
Limit theorems. Bearing capacity.

Statically admissible field of stresses is a distribution of stresses that satisfies the following conditions :

- it satisfies the conditions of equilibrium in each point of the body,
- it satisfies the boundary conditions for the stresses,
- the yield condition is not exceeded in any point of the body.

Lower bound theorem:
The true failure load is larger than the load corresponding to an equilibrium system.

$$p = 4c \rightarrow p \geq 4c$$

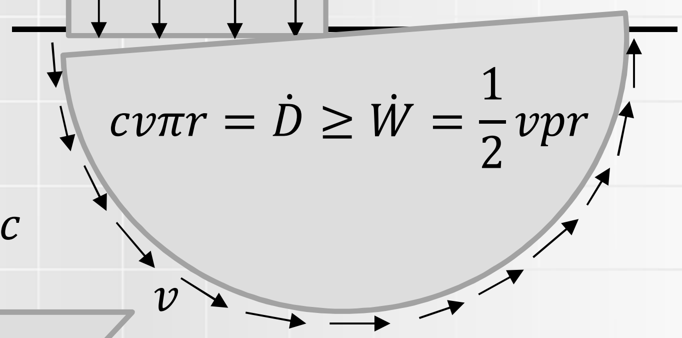


Kinematically admissible field of displacements is a distribution of displacements that satisfies the following conditions :

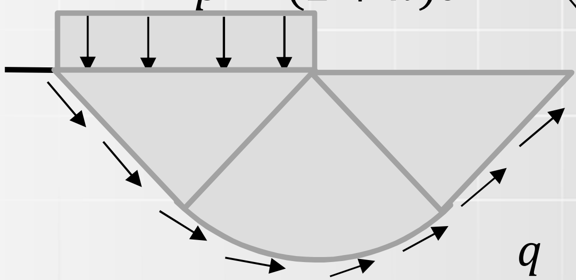
- the displacement field is compatible, i.e. no gaps or overlaps are produced in the body (sliding of one part along another part is allowed),
- it satisfies the boundary conditions for the displacements,

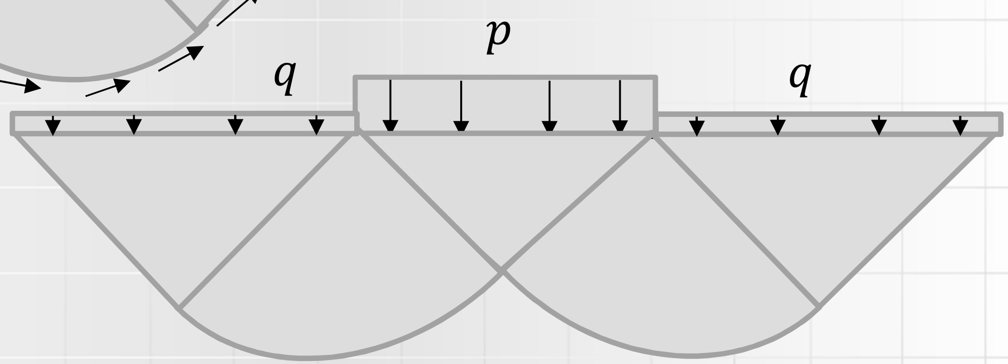
Upper bound theorem:
The true failure load is smaller than the load corresponding to a mechanism (if the latter) is determined using the virtual work principle.

$$p \leq 2\pi c \text{ (changing geometry: } 5.5c)$$



$$p = (2 + \pi)c$$



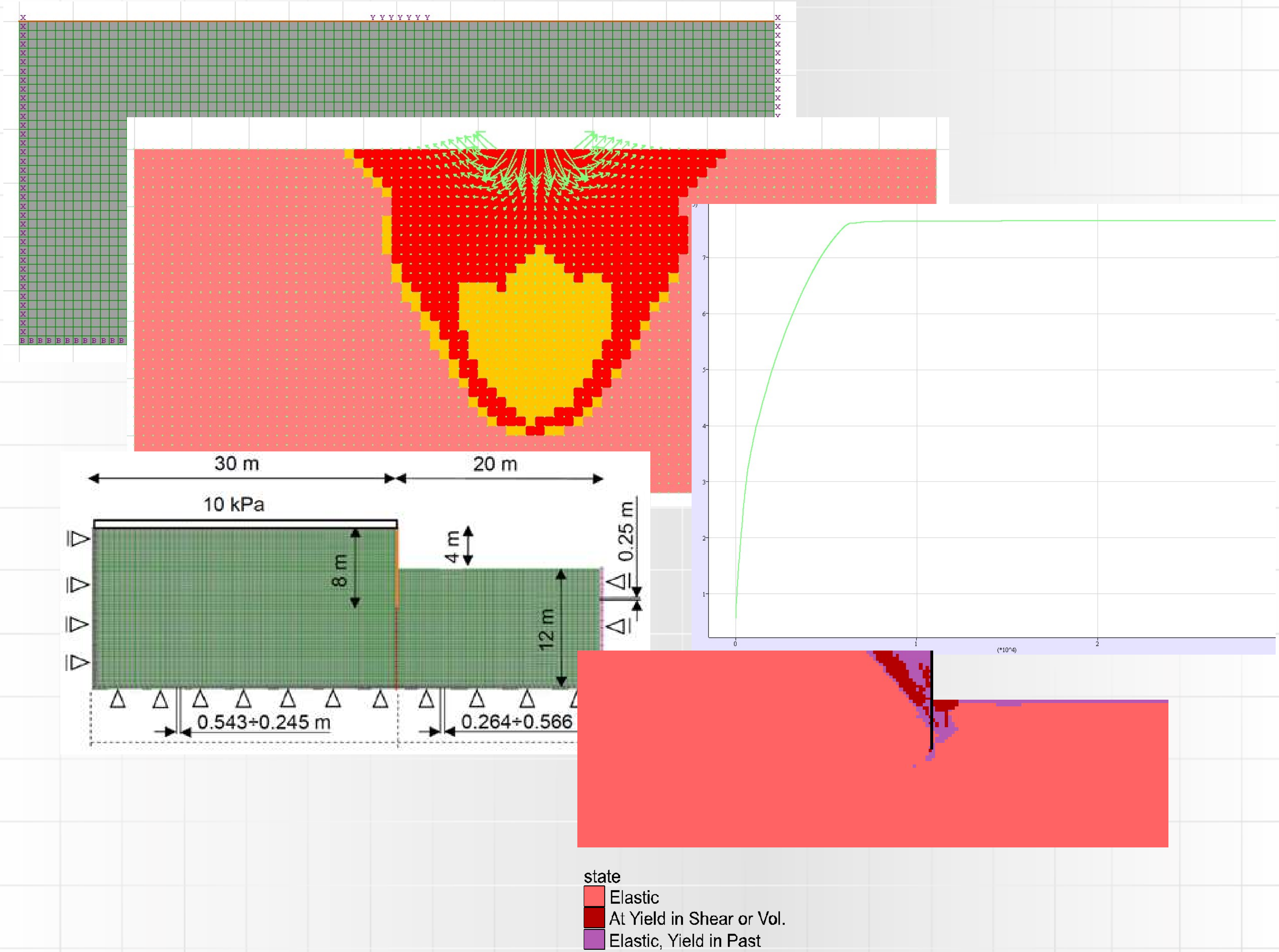


$$p = cNc + qNq + \frac{1}{2}\gamma D N_\gamma$$

$$N_q = \frac{1 + \sin\varphi}{1 - \sin\varphi}\exp(\pi \tan\varphi) \qquad N_c = (N_q - 1)\cot\varphi$$

$$N_\gamma \sim 2(N_q - 1)\tan\varphi$$

FEM, FDM, FELA


30 m
20 m
10 kPa
8 m
4 m
0.25 m
12 m
0.543÷0.245 m
0.264÷0.566
state
Elastic
At Yield in Shear or Vol.
Elastic, Yield in Past

Bibliography

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