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Soil Mechanics -Lecture III: Deformability. Consolidation.

HR EXCELLENCE IN RESEARCH

European Funds
Knowledge Education Development

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European Union
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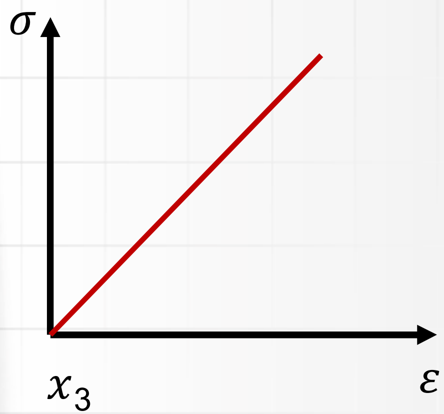
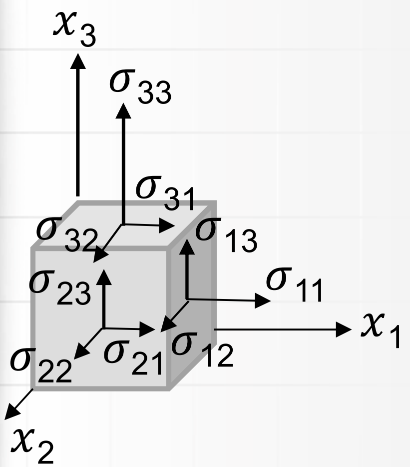
Deformability of soil. Elastic model.





Deformation tensor

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1} \qquad \varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2} \qquad \varepsilon_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right) \qquad \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial xj} + \frac{\partial u_j}{\partial xi}\right)$$

$$\varepsilon_{22} = \frac{\partial u_3}{\partial x_3} \qquad \varepsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right)$$

Physical relations: Hoek's law

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} \lambda + 2G & \lambda & \lambda & & & \\ \lambda & \lambda + 2G & \lambda & & 0 & \\ \lambda & \lambda & \lambda & & & \\ & & & 2G & 0 & 0 \\ & 0 & & 0 & 2G & 0 \\ & & & 0 & 0 & 2G \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{13} \\ \varepsilon_{23} \end{Bmatrix} \qquad \sigma_{ij} = C_{ijkl}\,\varepsilon_{ij}$$

Equation: equlibrium condition

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3} + F_1 = 0$$

$$\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + F_2 = 0 \qquad \frac{\partial \sigma_{ij}}{\partial xj} + Fi = 0$$

$$\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + F_1 = 0$$

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$$C_{ijkl} = \begin{bmatrix} K+\frac{4}{3}G & K-\frac{2}{3}G & K-\frac{2}{3}G & & & \\ K-\frac{2}{3}G & K+\frac{4}{3}G & K-\frac{2}{3}G & & 0 & \\ K-\frac{2}{3}G & K-\frac{2}{3}G & K+\frac{4}{3}G & & & \\ & & & 2G & 0 & 0 \\ & 0 & & 0 & 2G & 0 \\ & & & 0 & 0 & 2G \end{bmatrix}$$

$$K = \frac{E}{3(1-2\upsilon)}$$

$$G = \frac{E}{2(1+\upsilon)}$$

$$C_{ijkl}{}^{-1} = \begin{bmatrix} \frac{1}{E} & \frac{-\upsilon}{E} & \frac{-\upsilon}{E} & & & \\ \frac{-\upsilon}{E} & \frac{1}{E} & \frac{-\upsilon}{E} & & 0 & \\ \frac{-\upsilon}{E} & \frac{-\upsilon}{E} & \frac{1}{E} & & & \\ & & & \frac{1}{G} & 0 & 0 \\ & 0 & & 0 & \frac{1}{G} & 0 \\ & & & 0 & 0 & \frac{1}{G} \end{bmatrix}$$

$$\varepsilon_{11} = -\frac{1}{E}[\sigma_{11} - \upsilon(\sigma_{22} + \sigma_{33})]$$

$$\varepsilon_{22} = -\frac{1}{E}[\sigma_{22} - \upsilon(\sigma_{11} + \sigma_{33})]$$

$$\varepsilon_{33} = -\frac{1}{E}[\sigma_{33} - \upsilon(\sigma_{11} + \sigma_{22})]$$

$$\xrightarrow[\varepsilon_{11} = \varepsilon_{22} = 0]{Confined\ state} \quad \sigma_{22} = \sigma_{11} = \frac{\upsilon}{1-\upsilon}\sigma_{33}$$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}$$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2}$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)$$

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix} = \frac{E}{1-\upsilon^2}\begin{bmatrix}1 & \upsilon & 0\\ \upsilon & 1 & 0\\ 0 & 0 & (1-\upsilon)/2\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{Bmatrix}$$

$$\varepsilon_{33} = -\frac{\upsilon}{E}(\sigma_{11} + \sigma_{22})$$

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix} = \frac{E}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix}1-\upsilon & \upsilon & 0\\ \upsilon & 1-\upsilon & 0\\ 0 & 0 & (1-2\upsilon)/2\end{bmatrix}\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{Bmatrix}$$

$$\sigma_{33} = \frac{E\upsilon}{(1+\upsilon)(1-2\upsilon)}(\varepsilon_{11} + \varepsilon_{22})$$

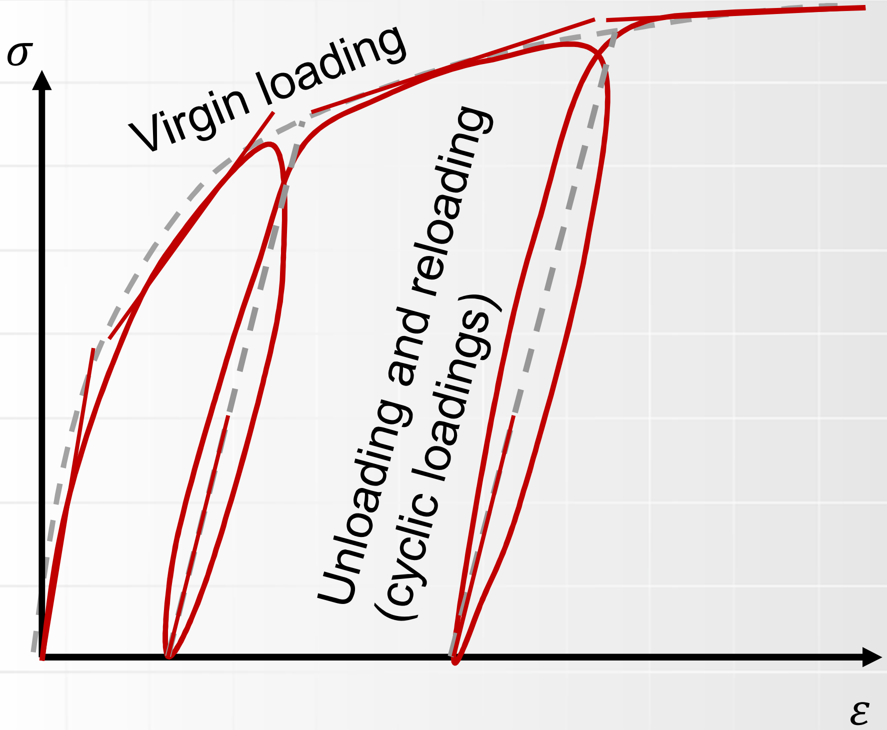
$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + F_1 = 0$$

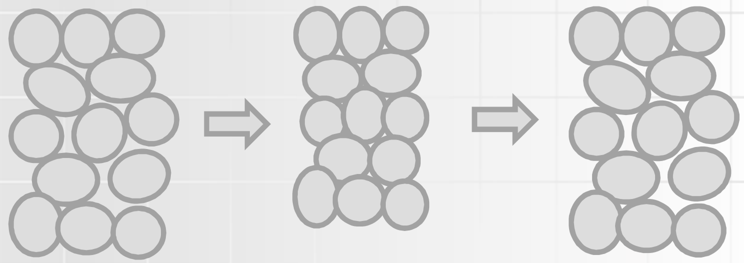
$$\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + F_2 = 0$$

$$\sigma_{11} = E\varepsilon_{11} \qquad \sigma_0 = -K\varepsilon_{vol} \qquad \sigma_{12} = -2G\varepsilon_{12}$$

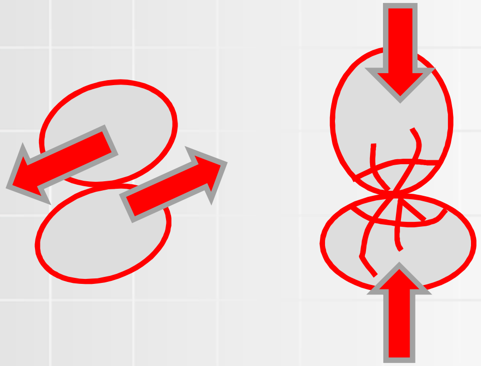
Anisotropy. Nonlinearity

Stress –strain dependency

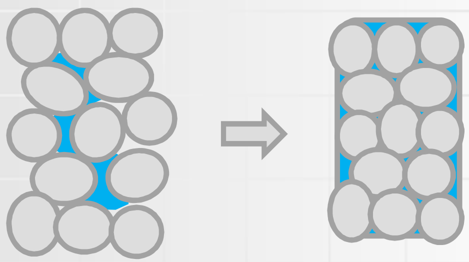




Changes in the packing system

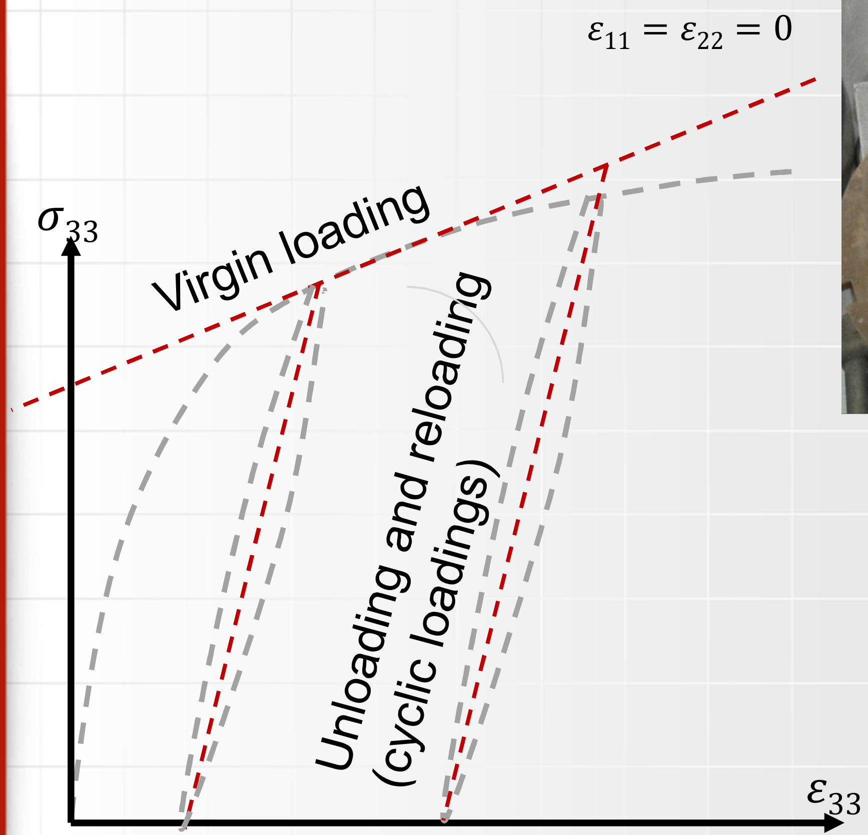


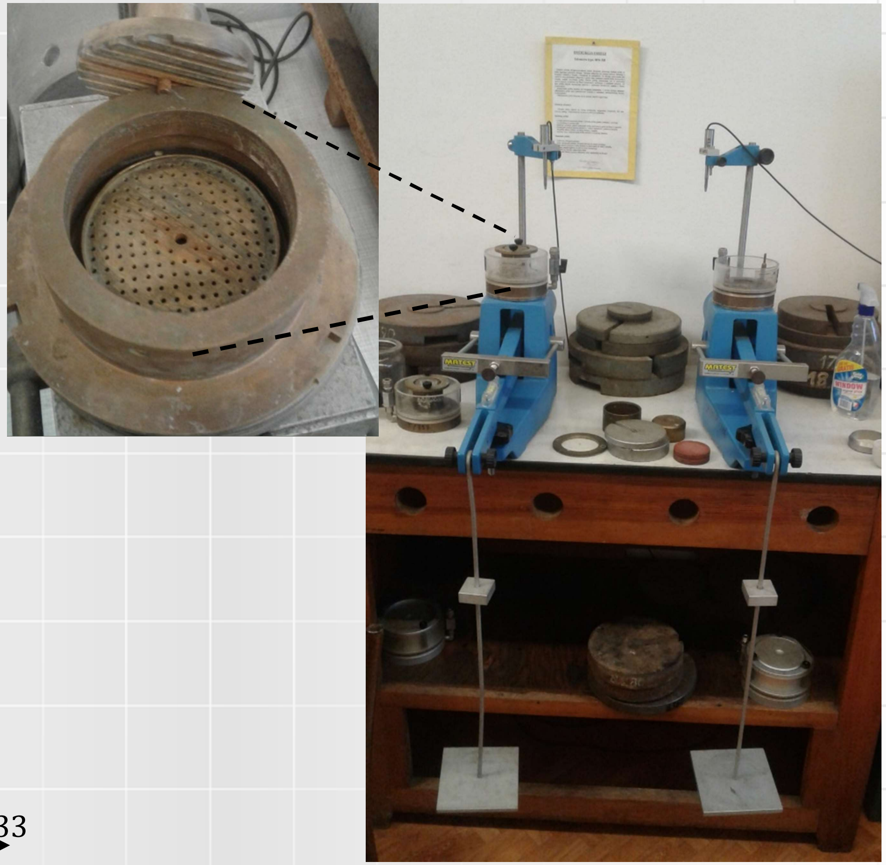
Shearing and contact forces



Influence of water

One-dimensional compression





Compression constant

$$\varepsilon = -\frac{1}{C}\ln\left(\frac{\sigma}{\sigma_1}\right) \qquad \varepsilon = -\frac{1}{C_{10}}\log\left(\frac{\sigma}{\sigma_1}\right) \qquad C_{10} = \frac{C}{2.3}$$

Confined modulus

$$\sigma_{22} = \sigma_{11} = \frac{v}{1-v}\sigma_{33} \qquad \sigma_{33} = \frac{E(1-v)}{(1+v)(1-2v)}\varepsilon_{33}$$

$$\sigma_{33} = E_{eod}\varepsilon_{33}$$

Is it just for unloading/reloading?

What about virgin loading?

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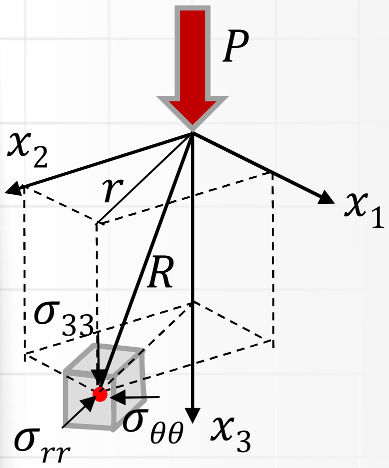
Boussinesq. Flamant.



$$\sigma_{33} = \frac{3P}{2\pi}\frac{x_3{}^3}{R^5}$$

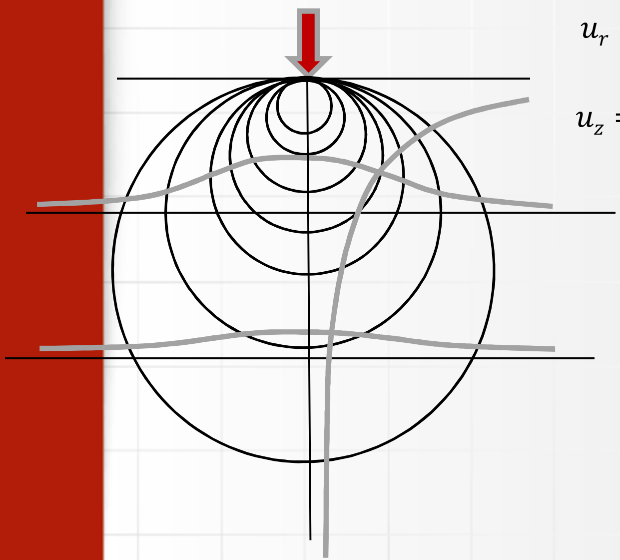
$$\sigma_{rr} = \frac{P}{2\pi}\left[\frac{3r^2x_3}{R^5} - (1-2\upsilon)\frac{1}{R(R+x_3)}\right]$$

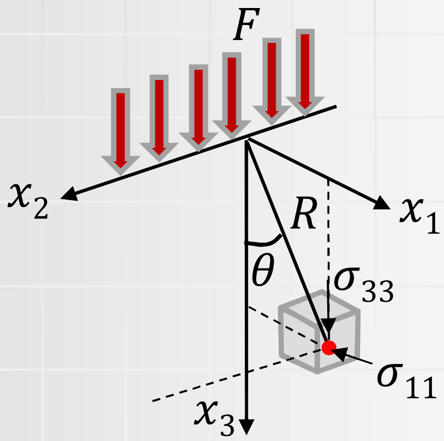
$$\sigma_{\theta\theta} = \frac{P}{2\pi}\frac{(1-2\upsilon)}{R^2}\left(\frac{R}{R+x_3} - \frac{x_3}{R}\right)$$

$$\sigma_{33} = \frac{3P}{2\pi}\frac{rx_3{}^2}{R^5}$$

$$u_r = \frac{P(1+\upsilon)}{2\pi ER}\left[\frac{r^2x_3}{R^3} - (1-2\upsilon)\left(1-\frac{x_3}{R}\right)\right]$$

$$u_z = \frac{P(1+\upsilon)}{2\pi ER}\left[2(1-\upsilon) + \frac{x_3}{R}\right]$$

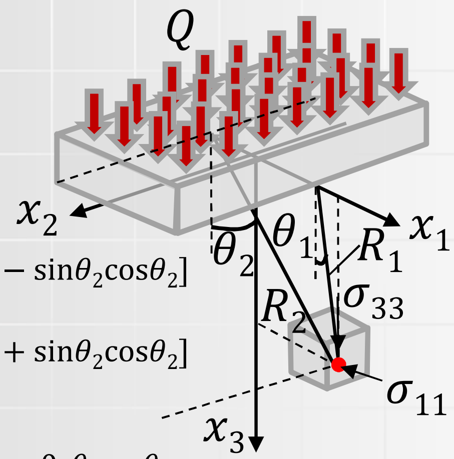




$$\sigma_{33} = \frac{2F}{\pi}\frac{x_3{}^3}{R^4} = \frac{2F}{\pi R}\cos^3\theta$$

$$\sigma_{11} = \frac{2F}{\pi}\frac{x_1{}^2\,x_3}{R^4} = \frac{2F}{\pi R}\sin^2\theta\,\sin\theta$$

$$\sigma_{13} = \frac{2F}{\pi}\frac{x_1\,x_3{}^2}{R^4} = \frac{2F}{\pi R}\sin\theta\,\cos^2\theta$$



$$\sigma_{33} = \frac{Q}{\pi}[(\theta_1-\theta_2) + \sin\theta_1\cos\theta_1 - \sin\theta_2\cos\theta_2]$$

$$\sigma_{11} = \frac{Q}{\pi}[(\theta_1-\theta_2) - \sin\theta_1\cos\theta_1 + \sin\theta_2\cos\theta_2]$$

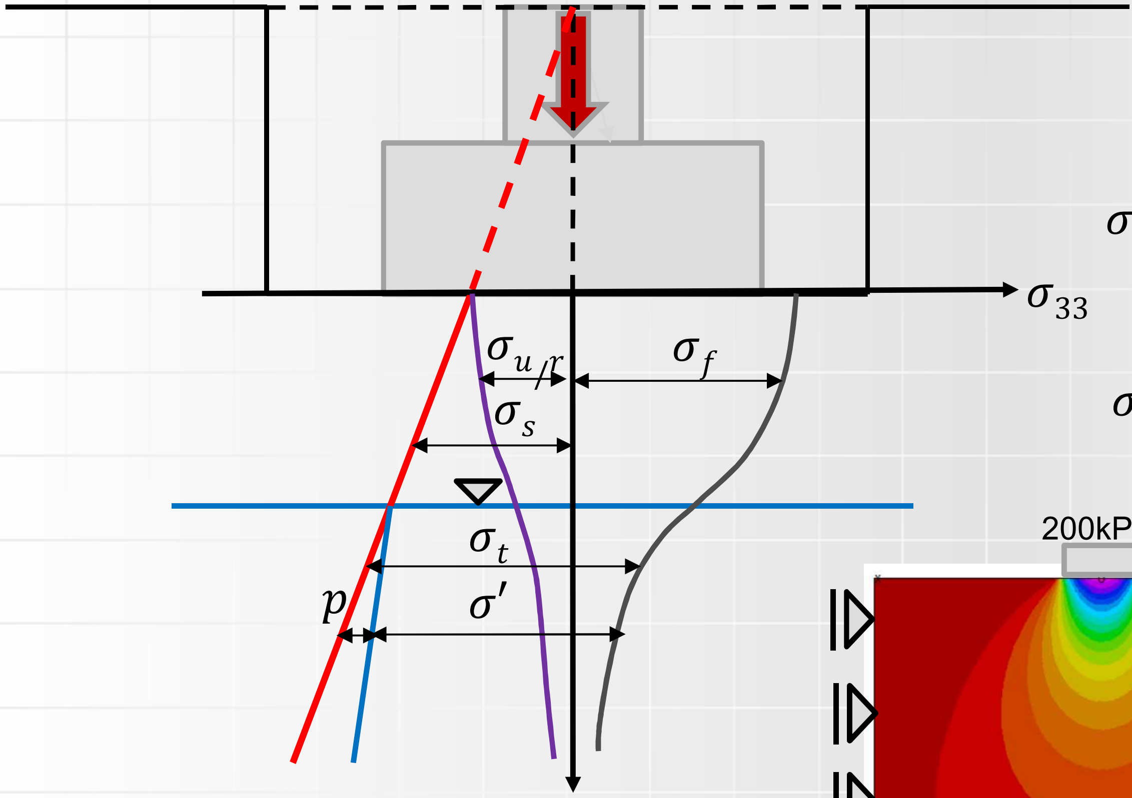
$$\sigma_{12} = \frac{Q}{\pi}[\cos^2\theta_2 - \cos^2\theta_1]$$

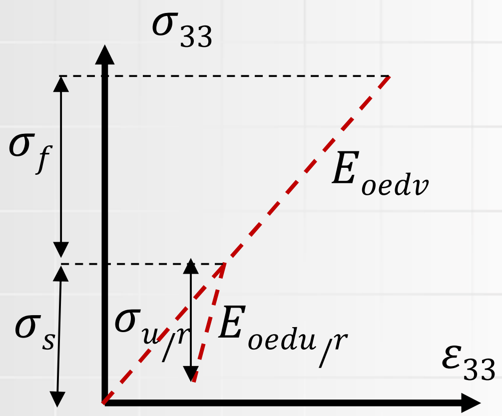
$$x_1 = 0, \theta_2 = \theta_1$$

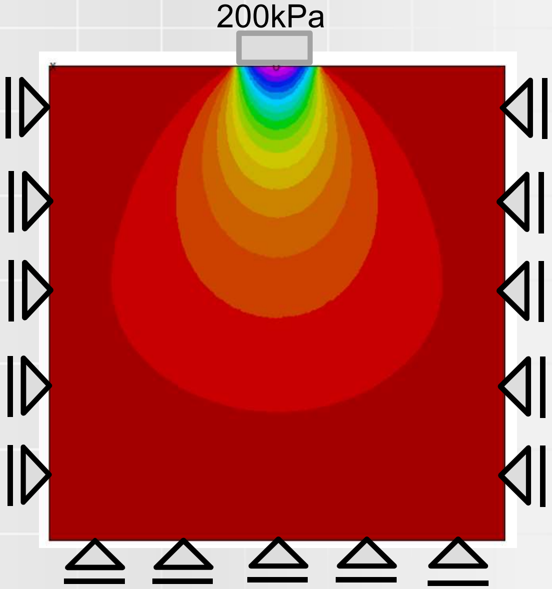
$$\sigma_{33} = \frac{2Q}{\pi}[\theta_1 + \sin\theta_1\cos\theta_1]$$

$$\sigma_{12} = 0$$

$$\sigma_{11} = \frac{2Q}{\pi}[\theta_1 - \sin\theta_1\cos\theta_1]$$







$$u = \int \frac{\sigma_{33}}{E_{oed}} dx_3 \approx \sum_{i=1}^{n} \frac{\sigma_{33av\,i}\, h_i}{E_{oed\,i}}$$

$$u_v = \sum_{i=1}^{n} \frac{\sigma_{33av\,i}\, h_i}{E_{oedv\,i}}$$

$$u_{u/r} = \sum_{i=1}^{n} \frac{\sigma_{33av\,i}\, h_i}{E_{oedu/r\,i}}$$

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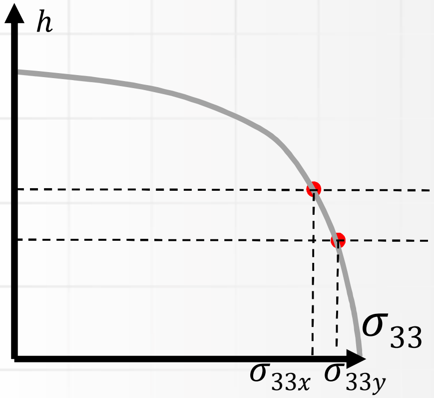
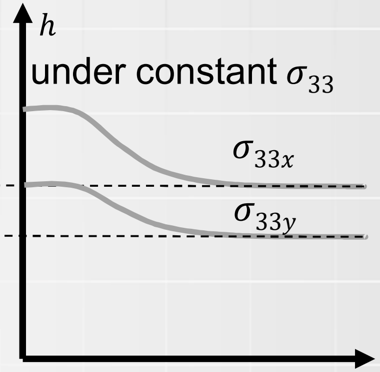
Consolidation





$$\Delta V_p = \varepsilon_{33}\Delta x_1 \Delta x_2 \Delta x_3$$

$$\Delta V_w = q_3 \Delta x_1 \Delta x_2 \Delta t$$

$$\varepsilon_{33}\Delta x_3 = q_3 \Delta t \rightarrow \frac{\partial \varepsilon_{33}}{\partial t} = \frac{\partial q_3}{\partial x_3}$$

$$\sigma_{33'} = Eeod\varepsilon_{33}$$

$$\varepsilon_{33} = \frac{1}{E_{eod}}(\Delta\sigma_{33} - p)$$

$$\frac{\partial \varepsilon_{33}}{\partial t} = \frac{1}{E_{eod}}\frac{\partial}{\partial t}(\Delta\sigma_{33} - p) = -\frac{1}{E_{eod}}\frac{\partial p}{\partial t}$$

$$q_3 = -\frac{k}{\gamma_w}\frac{\partial p}{\partial x_3}$$

$$\frac{\partial q_3}{\partial x_3} = -\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial {x_3}^2}$$

$$\frac{\partial p}{\partial t} = \frac{E_{eod}k}{\gamma_w}\frac{\partial^2 p}{\partial {x_3}^2}$$

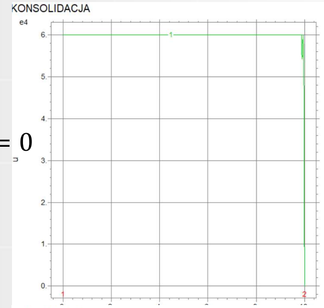
$$\frac{\partial p}{\partial t} = cv\frac{\partial^2 p}{\partial {x_3}^2}$$

$$t = 0, p = \Delta\sigma_{33}$$

$$t = \infty, p = 0$$

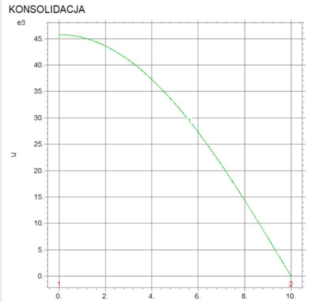
$t = 0$

$\frac{\partial p}{\partial x_3} = 0$

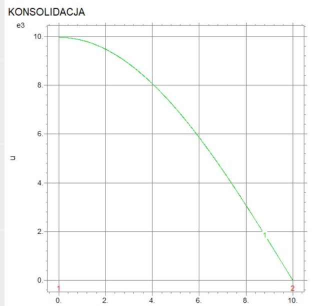


$p = 0$

$t = t_1$



$t = t_2$



Degree of consolidation

$$\varepsilon_{33} = \frac{1}{E_{eod}}(\Delta\sigma_{33} - p)$$

$$u = \Delta h = \int \frac{\sigma_{33} - p}{E_{oed}} dx_3 = \frac{\sigma_{33} h}{E_{oed}} - \int \frac{p}{E_{oed}} dx_3$$

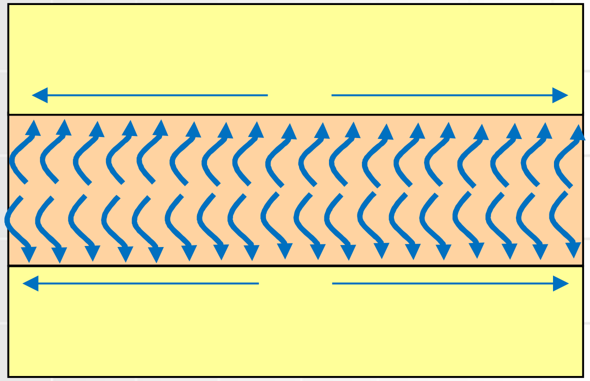
$$\Delta h_\infty = \frac{\sigma_{33} h}{E_{oed}}$$

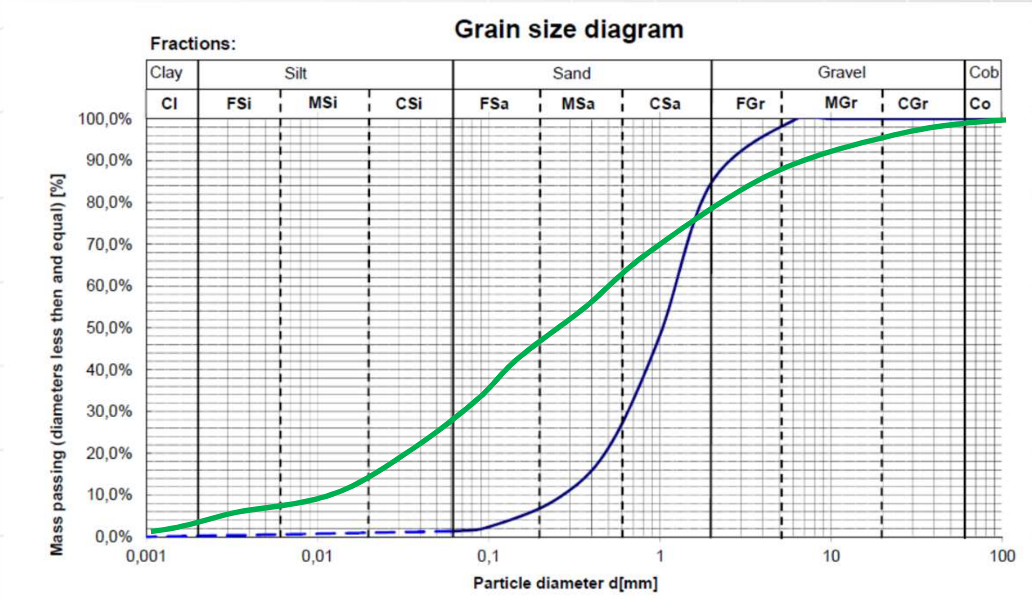
$$U = \frac{\Delta h - \Delta h_0}{\Delta h_\infty - \Delta h_0}$$

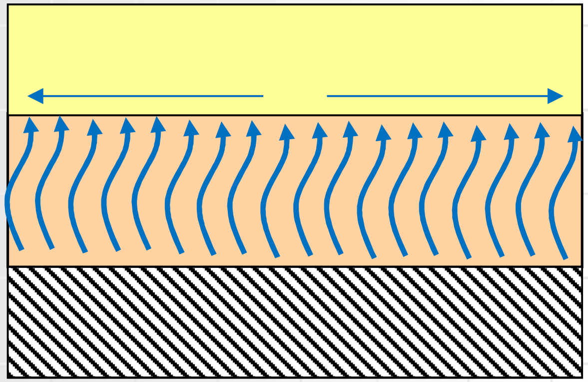
$$U = \frac{1}{h}\int_0^h \frac{p_0 - p}{p_0} dx_3$$

$$t_{99\%} = \frac{2h^2}{c_v} \qquad c_v = \frac{E_{eod} k}{\gamma_w}$$

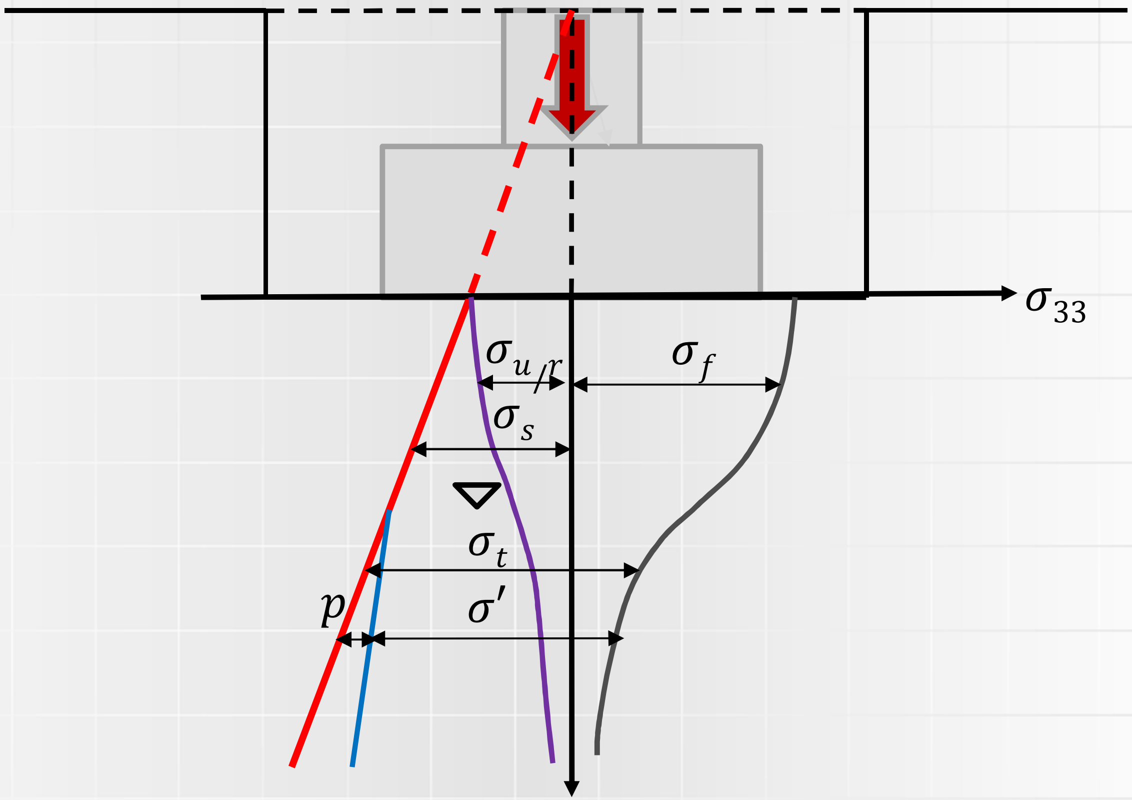
$$c_v = \frac{2h^2}{t_v} \rightarrow t_2 = \frac{t_1 {h_2}^2}{{h_1}^2}$$







Questions



- Elastic deformation and diplacement
- Virgin and lateral modulus
- Solving some 1-dimention consolidation problem
- Consolidation of opened and half-opened layer

Bibliography

Verruijt, A., & Van Baars, S. (2007). *Soil mechanics* (pp. 19-25). Delft, the Netherlands: VSSD.

http://www.padtinc.com/mae323/lecture4_plane_stress_and_singularities.pdf